AS.110.615, Johns Hopkins University

Algebraic Topology I

Fall Semester 2026

Lectures — MW 1:30–2:45 pm, Bloomberg 276 (Homewood campus)

Instructor — David Gepner, Krieger 410; office hours TBD, and by appointment

About the Course

This is the first semester of the graduate algebraic topology sequence. The guiding idea of the subject is to attach algebraic invariants — groups, modules, chain complexes — to topological spaces in a functorial way, and to use these invariants to distinguish spaces and to prove theorems inaccessible by point-set methods alone.

Prerequisites. The equivalent of one semester each of graduate abstract algebra and real analysis, including point-set topology. Prior exposure to the fundamental group (e.g., at the level of AS.110.413) is helpful but not assumed. Comfort with modules over a ring and basic categorical language will be assumed after the first two weeks.

Lectures

Complete course notes (Lectures 1–12 and appendices) · updated October 22, 2026.

Lecture	Dates	Topic
1	Aug 31 & Sep 2	Additive and Abelian Categories
2	Sep 9	Modules, Derived Categories, and the Tensor Product
3	Sep 14 & 16	Resolutions, Tor, and Ext
4	Sep 21 & 23	Simplicial Sets, Spaces, and Singular Chains
5	Sep 28 & 30	Generalized Homology and the Eilenberg–Steenrod Axioms
6	Oct 5 & 7	Homotopy Invariance and First Computations
7	Oct 12 & 14	Homotopy Pushouts and Mayer–Vietoris
8	Oct 19 & 21	Computations from the Eilenberg–Steenrod Axioms
9	Oct 26 & 28	Homotopy Groups and the Fundamental Group Action
10	Nov 2 & 4	Fibrations and the Homotopy Exact Sequence
11	Nov 9 & 11	The Fundamental Group and Its Applications
12	Nov 16 & 18	Covering Spaces and Subgroups of Free Groups

Syllabus

Download the course syllabus (PDF).

Part I — Categories and Homological Algebra

Week 1

August 31 and September 2. Categories, functors, and natural transformations. Zero objects, products, coproducts, and biproducts. Additive and abelian categories; kernels, cokernels, images, and exact sequences. Chain complexes, homology, and chain homotopies.

Week 2

September 9. The snake lemma and homology long exact sequences. Mapping cones; homotopy and derived categories. Tensor products, their universal property, and computation from presentations.

Week 3

September 14 and 16. Projective and injective modules; resolutions and their comparison. Derived functors, Tor, flatness, and long exact sequences. Ext groups, extensions, and computations over commutative and noncommutative rings.

Week 4

September 21 and 23. Presheaves and the Yoneda lemma; the simplex category, simplicial sets, faces, and degeneracies. Geometric realization, singular simplices, and the realization–singular adjunction. Simplicial chain complexes, singular chains, and coefficients.

Part II — Singular Homology

Week 5

September 28 and 30. Homotopy invariance, the homotopy category, and homotopy pushouts. Generalized homology and the Eilenberg–Steenrod axioms in Mayer–Vietoris form; coefficients and the dimension axiom. Relative and reduced homology, homotopy cofibers, suspension, and the homology of spheres.

Week 6

October 5 and 7. The prism operator and the proof of homotopy invariance of singular homology. Homology of points, disjoint unions, and contractible spaces; deformation retractions. Relative chains, the long exact sequence of a pair, relative homotopies, and reduced homology.

Week 7

October 12 and 14. Double mapping cylinders, cofibrations, subdivision, and chain models of homotopy pushouts. Preservation of homotopy pushouts by singular chains with coefficients in an abelian group A. Mayer–Vietoris, excision, the gluing axiom, cell attachments, and mapping-cone exact sequences.

Week 8

October 19 and 21. Finite cell complexes, attaching maps, and degree. Cellular homology and comparison with singular homology; product cells and the product boundary formula. Homology of products of spheres, tori, and real and complex projective spaces. Mapping tori, the Wang sequence, and the Klein bottle.

Part III — Homotopy Theory and the Fundamental Group

Week 9

October 26 and 28. Homotopy groups, basepoints, and the action of the fundamental group. Statements of the Whitehead and Hurewicz theorems; proofs deferred to Math 616.

Week 10

November 2 and 4. Fibrations and the homotopy lifting property; Serre fibrations and fiber bundles. The long exact sequence of a fibration, the path–loop fibration, and the Hopf fibration; computation of the third homotopy group of the 2-sphere.

Week 11

November 9 and 11. The fundamental group of the circle via lifting; winding number and degree. Free products, relations, and the Seifert–van Kampen theorem. Fundamental groups of graphs, cell attachments, and surfaces. Applications to the fundamental theorem of algebra and Borsuk–Ulam in dimensions one and two.

Week 12

November 16 and 18. Covering spaces: unique lifting of paths and homotopies, the lifting criterion for maps, and classification of connected covers. Universal covers, deck transformations, and the Galois correspondence. Covering graphs, subgroups of free groups, and finite-index rank calculations. Remaining class time is reserved for review and project preparation.

Final Project Presentations

Nov 30–Dec 9
In-class final project presentations. No new lecture material.

The schedule from October 12 onward is provisional. Thanksgiving break: no classes during the university break.

Grading

Your course grade is based entirely on class participation/attendance and a final project. There are no exams.

1. **Participation and attendance (10%).** This is a lecture course, but a conversational one: attend regularly, ask and answer questions, and occasionally present an argument at the board when invited. The participation grade reflects consistent, good-faith engagement over the semester — not the number of times you speak, and certainly not whether your questions are "good enough." If you must miss more than an occasional class (illness, travel, conflicts), just let me know.
2. **Final project (90%).** An in-class presentation on a topic extending the course, to take place during the last two weeks of class (Nov 30 – Dec 9). There is no written paper. Suitable topics include: the Dold–Kan correspondence in full; model structures on simplicial sets; group cohomology via derived functors; the Hurewicz theorem; classifying spaces and principal bundles; Poincaré duality; the Lefschetz fixed point theorem; covering space theory from the Galois-theoretic viewpoint; braid groups and configuration spaces. Topic proposal due mid-October; optional draft of your presentation materials early November. Graded on mathematical correctness, depth of engagement with the material, and clarity of exposition — not originality of research.
3. **Homework (assigned, not counted toward the grade).** One problem set per week, graded for your benefit only and returned with comments. Collaboration is encouraged; write up solutions in your own words.

Homework

Week	Dates	Topic
1	Aug 31 & Sep 2	Categories and Chain Complexes
2	Sep 9	Homotopies, Exact Sequences, and Cones
3	Sep 14 & 16	Derived Functors, Tor, and Ext
4	Sep 21 & 23	Simplicial Sets and Singular Chains
5	Sep 30	Generalized Homology and the Eilenberg–Steenrod Axioms
6	Oct 5 & 7	Homotopy Invariance
7	Oct 12 & 14	Homotopy Pushouts and Homology
8	Oct 19 & 21	Cellular Homology and Mapping Tori
9	Oct 26 & 28	Homotopy Groups and the Fundamental Group Action
10	Nov 2 & 4	Fibrations and Homotopy Exact Sequences
11	Nov 9 & 11	Fundamental Groups and Attaching Maps
12	Nov 16 & 18	Covering Spaces and Free Subgroups

Homework 9–12 in one PDF. Dates above identify the associated lecture weeks; no new homework due dates are assigned here.

Algebraic Topology II — Draft Materials

Provisional second-course plan and opening notes on generalized cohomology, spectra, and spectral sequences. These are drafts, not a complete second course. The plan proposes where existing appendix material could be integrated; all appendices remain in the current Math 615 notes.

Important Dates

Date	
Mon, Aug 31	First class
Mid-October	Project topic proposal due
Early Nov	Optional draft of presentation materials
Late Nov	Thanksgiving break
Nov 30 – Dec 9	Final project presentations (last two weeks of class); last class Wed, Dec 9

Policies

1. **Collaboration and integrity.** Discussion of homework with classmates is encouraged. The final project must be your own exposition; all sources (including AI tools, if used) must be cited. All work is subject to the university's graduate academic misconduct policies.
2. **AI use.** Prohibited on homework: the problem sets exist entirely for your benefit, so outsourcing them defeats the purpose. Permitted for everything else, subject to the citation requirement for the final project.
3. **Devices in class.** Discouraged. If your attention drifts, work on something productive instead — a lit screen makes it harder for everyone nearby to pay attention.
4. **Accommodations.** Students requiring accommodations should register with Student Disability Services and contact me early in the semester.

Texts

No single text covers the course in the order we follow. Primary references, all freely or electronically available:

1. A. Hatcher, *Algebraic Topology* — free from the author's page. Main reference for homology, the fundamental group, and covering spaces.
2. C. Weibel, *An Introduction to Homological Algebra* — Cambridge University Press; main reference for Part I.
3. S. Gelfand and Y. Manin, *Methods of Homological Algebra* — SpringerLink; for derived and triangulated categories.
4. P. Goerss and J. F. Jardine, *Simplicial Homotopy Theory* — SpringerLink; main reference for simplicial sets.
5. J. P. May, *A Concise Course in Algebraic Topology* — free PDF from the author's page; secondary reference throughout, especially for fibrations.

AS.110.615, Department of Mathematics, Johns Hopkins University. Questions about the course: gepner@jhu.edu. Page maintained by the instructor; last updated October 22, 2026.